Problem 7.9

A force acts on a particle over a displacement. Both are characterized in unit vector notion below.

$$\vec{F} = \left(6\hat{i} - 2\hat{j}\right)\text{ N} \quad \text{and} \quad \Delta\vec{r} = \left(3\hat{i} + \hat{j}\right)\text{ m}$$

a.) How much work does the force due during the displacement?

We need to start with:

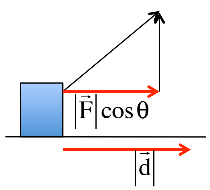
$$W_F = \vec{F} \bullet \vec{d}$$

The temptation is to use the Pythagorean relationship to determine the magnitude of "F" and the magnitude of "r," then try to figure out the angle between the two so you can use $W_F = |\vec{F}||\vec{d}|\cos\phi$ to determine the work done. Actually, in this case, that wouldn't be all that easy as the angle calculation isn't really straight forward.

So how might you do it? Well, what do you know about *dot products*? Maybe we could be clever . . .

$$\vec{F} = \left(6\hat{i} - 2\hat{j}\right)\text{ N} \quad \Delta\vec{r} = \left(3\hat{i} + \hat{j}\right)\text{ m}$$

By definition, at least on a conceptual level, a *dot product* produces the product of the magnitude of one vector ("$|\vec{d}|$" in the sketch to the right) and the magnitude of the component of the second vector IN THE DIRECTION OF THE FIRST VECTOR ("$|\vec{F}|\cos\theta$" in the sketch to the right). That product gives you "$|\vec{F}||\vec{d}|\cos\theta$," which by definition is the work done by "F."



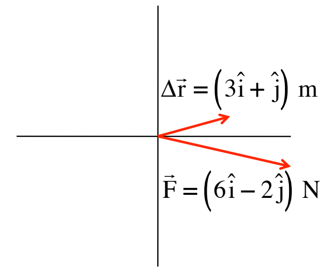
It follows that if the two vectors are presented in a unit vector notation, the *dot product* will just be the product of the like components, added together, or:

$$\vec{F} \bullet \vec{d} = \left(F_x d_x + F_y d_y + F_z d_z\right)$$

Using this with our vectors, we can write:

$$\begin{aligned}
\vec{F} \bullet (\Delta\vec{r}) &= F_x(\Delta r)_x + F_y(\Delta r)_y + F_z(\Delta r)_z \\
&= (6.00\text{ N})(3.00\text{ m}) + (-2.00\text{ N})(1.00\text{ m}) + (0\text{ N})(0\text{ m}) \\
&= (18.0\text{ N}\bullet\text{m}) - (2.00\text{ N}\bullet\text{m}) \\
&= 16.0\text{ J}
\end{aligned}$$

b.) Getting the angle by graphing the "F" and "r" vectors, then by being clever with trig, that might work, but it seems awkward. Another possibility is to use what you know about *work* calculations coupled with what you know about *this* situation. Doing that yields:



$$|\vec{F}| \qquad\qquad |\vec{d}| \qquad\qquad \cos\theta = 16.0\text{ J}$$

$$\left[(6.00\text{ N})^2 + (-2.00\text{ N})^2\right]^{1/2}\left[(3.00\text{ m})^2 + (1.00\text{ m})^2\right]^{1/2}\cos\theta = 16.0\text{ J}$$

$$\Rightarrow \qquad \theta = 36.9^\circ$$